gyro rotors are governed by Eqs. (51); whereas, the simple theory curves were generated by assuming that the spin-axis directions remain fixed in-inertial space, that is, $\mathbf{m}_i = \mathbf{n}_i$ $(i = 1,2,3)$.

It is seen that the full theory accomplishes its intended purpose; that is, θ_1, θ_2, and θ_3 approach zero asymptotically. Use of the simple theory, on the other hand, leads to oscillatory behavior of θ_1 and θ_2. Since oscillatory behavior of θ_1, θ_2, and θ_3 occurs in the absence of any control torque, it thus appears that the use of simple theory can produce pitch control, but is of little value as regards roll and yaw control. For a control scheme of the kind under consideration, the use of full theory therefore leads to significant improvements in performance.

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Thrust Profile Shaping for Spin-Stabilized Vehicles

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Analytical results are presented which show that the wobble motion and average pointing error caused by large thrust-line offsets can be greatly reduced by employing a shaped rocket thrust profile. A simplified analytical procedure is developed to evaluate the effect of thrust shape on the wobble motion and the average pointing error during the thrust and post-thrust periods. Performance relative to a nonshaped rectangular profile is conveniently expressed in terms of *K*-factors which depend on the Laplace transform of the shaped profile. This formulation shows that the relative pointing error *K*-factor is of the same form as the wobble motion *K*-factor with the spin rate ω replacing the polhode rate Ω. Thrust rise and decay shaping effects are presented for elementary time functions in terms of generalized parameters. The results provide a simplified basis for the preliminary specification of thrust shaping requirements and for the evaluation of candidate rocket motor designs.

Nomenclature

x, y, z = principal body axes (x-axis longitudinal)
ω, q, r = angular velocities about x, y, and z axes, respectively
I_x, I = moments of inertia about x-axis and transverse axes (y, z), respectively
ϕ, θ, ψ = roll, pitch, yaw Euler angles of body axes with respect to inertial axes
Ω = $(I - I_x)\omega/I$, polhode rate
ω_{yz} = $q + ir$, transverse rate
α = $\theta + i\psi$, inertial pointing angle
$N(t)$ = disturbance torque about z-axis
N = maximum value of $N(t)$
$n(t)$ = $N(t)/N$
β = frequency of sinusoidal thrust profiles
$K_{\tilde{\omega}_{yz}}, K_{\bar{\alpha}}$ = thrust shaping performance factors for oscillatory transverse rates and average pointing errors, respectively (relative to a rectangular profile)

Subscripts

o, f = evaluated at thrust onset, thrust termination, respectively
r, d = evaluated at end of thrust rise, beginning of thrust decay, respectively

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Introduction

SPIN stabilization performance during vacuum thrusting maneuvers has generally been analyzed for constant body-fixed torques corresponding to a nonshaped rectangular thrust-time profile. The interaction between thrust shape and spin dynamics appears to have received little attention in the literature.[1–7] Papis[1] investigated the interaction for a ramp input based upon a concern about the discrepancy between the finite thrust rise-time of actual rocket motors and the instantaneous rise of the rectangular thrust model.

The concept of deliberately shaping the thrust profile in order to enhance spin stabilization performance evolved from applications with a large thrust-line offset and a small difference between the roll and transverse inertias. The wobble motion (oscillatory transverse rate) caused by the thrust-line offset during thrust rise and decay is inversely proportional to the product of the inertia difference $I - I_x$ and the spin rate, ω. Large wobble motions cause a loss in the imparted velocity increment (cosine loss) and may seriously degrade post-thrust attitude stability if the vehicle is de-spun to a low-roll rate after thrust completion. The average pointing error is less difficult to control in such applications since it is inversely proportional to the *square* of the spin rate. Thrust profile shaping via solid motor propellant grain design (or combinations of propellant segments with different burning rates) appears to be an attractive method of control for both the oscillatory transverse rate and the average pointing error. This paper presents an analytical basis for the design of a thrust-shaped spin stabilization system.

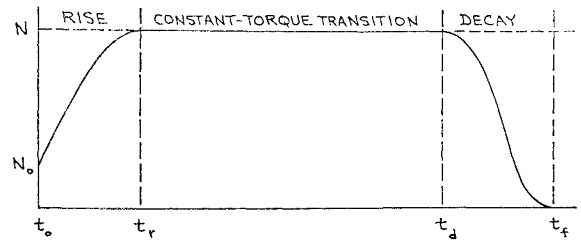

Fig. 1 Disturbance torque profile.

Thrust Shaping Analysis

The analysis is simplified by considering the thrust rise shaping separately from the thrust decay shaping through the artifice of a constant-thrust transition as shown in Fig. 1. A body-fixed yaw disturbance torque $N(t)$ is assumed corresponding to a fixed Y-offset and a time-varying thrust profile. The equations of motion for a spinning symmetrical vehicle can be written as

$$\dot{\omega}_{yz} + i\Omega\omega_{yz} = iN(t)/I \tag{1}$$

(The polhode rate Ω is the precession rate of the angular velocity vector in body axes for the torque-free case.) For small angles the kinematical equation for the inertial pointing angle is

$$\dot{\alpha} = \omega_{yz}e^{i\phi} \tag{2}$$

where

$$\phi = \phi_o + \omega t$$

The rise and decay intervals are assumed to be small relative to the total burn time. The post-rise motion can be determined from a complete time-domain solution[1] for a specific thrust rise profile $N(t)$. However, significant physical insight and computational advantages are obtained by extracting the parameters of interest from a Laplace transform analysis. Taking the Laplace transform of Eq. (1) yields

$$\omega_{yz}[s] = (iN[s]/I + \omega_{yz_o})/(s + i\Omega) \tag{3}$$

The transverse rate for the post-rise constant-torque phase consists of a constant term $\bar{\omega}_{yz}$ and an oscillatory term $\tilde{\omega}_{yz}\exp(-i\Omega t)$. Therefore, the partial fraction expansion of Eq. (3) must be of the following form

$$\omega_{yz}[s] = \bar{\omega}_{yz}/s + \tilde{\omega}_{yz}/(s + i\Omega) + \text{other terms} \tag{4}$$

The coefficients of interest can be readily evaluated as follows:

$$\bar{\omega}_{yz} = s\omega_{yz}[s]|_{s=0} = \{s(iN[s]/I + \omega_{yz_o})/(s + i\Omega)\}|_{s=0} \tag{5}$$

Since $N(t)$ is constant for $t > t_r$, the partial fraction expansion of $N[s]$ is

$$N[s] = N/s + \text{other terms}$$

Therefore,

$$\bar{\omega}_{yz} = N/I\Omega = N/(I - I_x)\omega \tag{6}$$

Similarly,

$$\tilde{\omega}_{yz} = \left.\frac{(s + i\Omega)(iN[s]/I + \omega_{yz_o})}{s + i\Omega}\right|_{s=-i\Omega} = \omega_{yz_o} + \left.\frac{iN[s]}{I}\right|_{s=-i\Omega} \tag{7}$$

The post-rise solution is

$$\omega_{yz} = \bar{\omega}_{yz} + \tilde{\omega}_{yz}e^{-i\Omega t},\ t_r < t < t_d \tag{8}$$

where the complex coefficients are given by Eqs. (6) and (7). The post-rise equilibrium transverse rate $\bar{\omega}_{yz}$ provides the gyroscopic coupling that balances the steady disturbance. For zero initial conditions, the post-rise oscillatory transverse rate $\tilde{\omega}_{yz}$ is a measure of the wobble induced by the thrust-rise disturbance. If $N(t)$ is a step input

$$\tilde{\omega}_{yz}(\text{step}) = iN/sI|_{s=-i\Omega} = -N/(I - I_x)\omega \tag{9}$$

Thrust shaping performance relative to the step input profile is therefore,

$$K_{\tilde{\omega}_{yz}} = \tilde{\omega}_{yz}/\tilde{\omega}_{yz}(\text{step}) = sN[s]/N|_{s=-i\Omega} \tag{10}$$

where the K-factor denotes the magnitude of the complex number. For the instantaneous rise, the K-factor is unity.

The intertial motion is determined in a similar manner. The Laplace transform of Eq. (2) for $\phi_o = 0$ is

$$s\alpha[s] - \alpha_o = \int_0^\infty \omega_{yz}(t)e^{i\omega t}e^{-st}\,dt = \omega_{yz}[s - i\omega] \tag{11}$$

However, from Eq. (3)

$$\omega_{yz}[s - i\omega] = (iN[s - i\omega]/I + \omega_{yz_o})/(s - i\omega + i\Omega) \tag{12}$$

Therefore,

$$\alpha[s] = (iN[s - i\omega]/I + \omega_{yz_o})/s(s - i\omega(I_x/I)) + \alpha_o/s \tag{13}$$

The post-rise attitude motion during the constant-torque phase consists of a constant term and oscillatory terms with frequencies of ω and $\omega(I_x/I)$. (This form, commonly referred to as "tricyclic" motion, can be inferred from Eq. (13) with $N[s] = N/s$ for the constant-torque phase.) Therefore, the partial fraction expansion of $\alpha(s)$ must be of the following form,

$$\alpha[s] = \bar{\alpha}/s + \tilde{\alpha}/(s - i\omega(I_x/I)) + \hat{\alpha}/(s - i\omega) + \text{other terms} \tag{14}$$

The coefficients of interest are determined using Eqs. (13) and (14) in a manner analogous to the previous development. The results are

$$\bar{\alpha} = \alpha_o + iI\omega_{yz_o}/I_x\omega - N[s]/I_x\omega|_{s=-i\omega} \tag{15}$$

$$\tilde{\alpha} = -iI\tilde{\omega}_{yz}/I_x\omega \tag{16}$$

$$\hat{\alpha} = -iN/(I - I_x)\omega^2 \tag{17}$$

The post-rise attitude is then

$$\alpha = \bar{\alpha} + \tilde{\alpha}e^{i\omega(I_x/I)t} + \hat{\alpha}e^{i\omega t} \tag{18}$$

The angle $\bar{\alpha}$ is the post-rise average pointing error; $\tilde{\alpha}$ is the wobble coefficient; and $\hat{\alpha}$ is the equilibrium coning angle due to the torque-induced shift in the effective spin axis and is not affected by shaping. The pointing error for the step input (instantaneous rise) with zero initial conditions is

$$\bar{\alpha}(\text{step}) = -iN/I_x\omega^2 \tag{19}$$

The performance of a shaped profile relative to the instantaneous rise can be written as

$$K_{\bar{\alpha}} = \bar{\alpha}/\bar{\alpha}(\text{step}) = sN[s]/N|_{s=-i\omega} \tag{20}$$

This formulation for thrust shaping performance shows that the relative pointing error factor $K_{\bar{\alpha}}$ is of the same form as the oscillatory transverse rate factor $K_{\tilde{\omega}_{yz}}$ with spin rate ω replacing polhode rate Ω. Therefore, only a single plot of the K-factor variation with frequency is needed for a given rise profile. Furthermore, the computation of K-factors by Eqs. (10) and (20) requires much less effort than the usual approach of deriving a complete time-domain solution for body rates and inertial angles.

Post-thrust residual motion is also of interest in many applications. The effect of thrust decay on residual motion can be isolated from rise effects by considering the body to be in equilibrium with zero pointing error at $t = t_d$

$$\bar{\omega}_{yz} = \frac{N}{(I - I_x)\omega}$$

$$\hat{\alpha} = -\frac{iN}{(I - I_x)\omega^2} \tag{21}$$

$$\tilde{\omega}_{yz} = \tilde{\alpha} = \bar{\alpha} = 0$$

The decay transient generates wobble motion $\tilde{\omega}_{yz}$ and causes a shift in the average pointing direction $\bar{\alpha}$. The magnitude of these terms for an instantaneous decay are the same as the values given in Eqs. (9) and (19). Relative thrust shaping performance is again expressed in terms of K-factors. It can be shown that the normalized K-factor data for a given rise profile is equally applicable to a decay profile obtained by reflecting the rise profile about a vertical line. The parameters N_o and $\dot{N}_o$ correspond to N_f and $-\dot{N}_f$.

Actually the interaction between rise and decay depends on the precise time relationship of the phases. The post-thrust residual motion consists of the vectorial combination of rise and decay effects as determined by the actual phase relations at $t = t_d$. Generally, worst case phasing or a statistical phase-angle variable is assumed for design purposes since several cycles of motion may occur between rise and decay.

Thrust Profile Factors

The K-factors are developed for sinusoidal, exponential and linear thrust-time profiles. These elementary shapes are compared in Fig. 2 on the basis of identical maximum slopes. Typically, the "initial" level at thrust onset N_o is modelled as a nonzero value for rise shapes, whereas the level at thrust burnout N_f is considered to be zero for the decay shapes. The normalized disturbance torque $n(t)$ is used for convenience. Thrust-rise performance data are shown in Figs. 3 and 4; thrust decay data are shown in Fig. 5.

Sinusoidal Quarters-Cycle Rise (Decay)

The K-factor computational procedure is illustrated for this case. The normalized time function is

$$n(t) = n_o + (1 - n_o)\sin\beta t,\ t \leq \pi/(2\beta)$$
$$n(t) = 1,\ t > \pi/(2\beta) \tag{22}$$

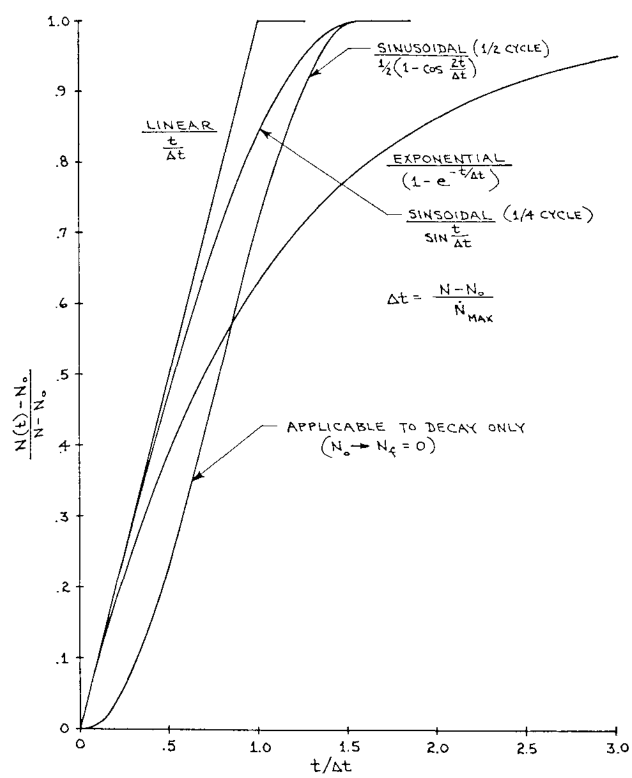

Fig. 2 Profile comparison for identical maximum slopes.

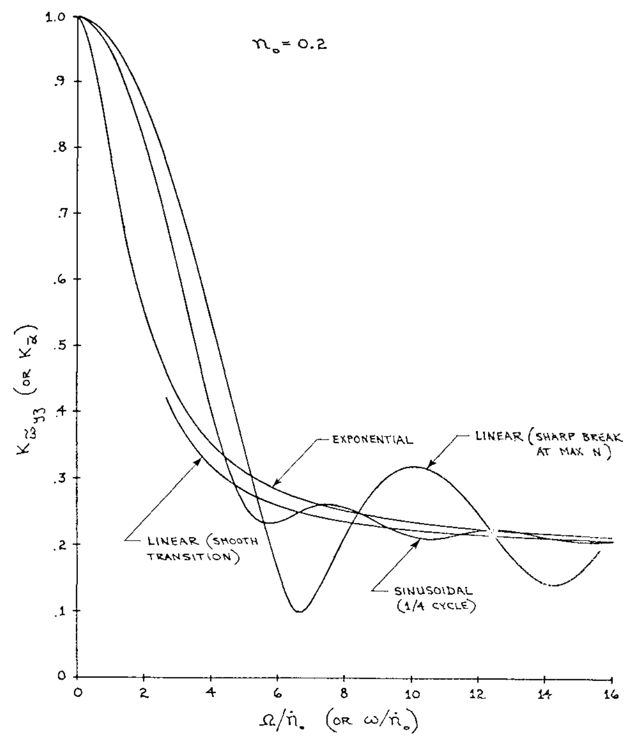

Fig. 3 Shaping effects during thrust rise.

The Laplace transform is computed as follows

$$n[s] = \int_0^{\pi/(2\beta)} [n_o + (1 - n_o)\sin\beta t]e^{-st}\,dt + \int_{\pi/(2\beta)}^{\infty} e^{-st}\,dt \tag{23}$$

The result is

$$n[s] = \frac{n_o}{s} + \frac{\beta(1 - n_o)[s + \beta e^{-\pi s/(2\beta)}]}{s(s^2 + \beta^2)} \tag{24}$$

The transverse rate factor is therefore

$$K_{\tilde{\omega}_{yz}} = sn[s]\,|_{s=-i\Omega}$$
$$= n_o + \frac{(1 - n_o)[i(\Omega/\beta) - e^{i\pi\Omega/(2\beta)}]}{(\Omega/\beta)^2 - 1} \tag{25}$$

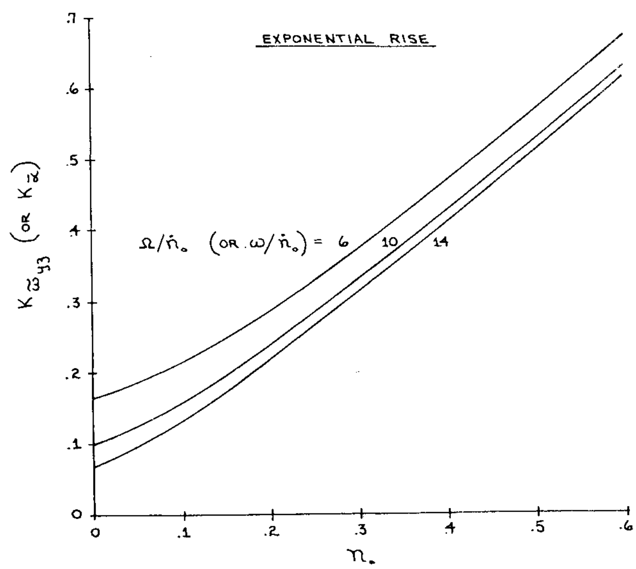

Fig. 4 Effect of initial thrust level.

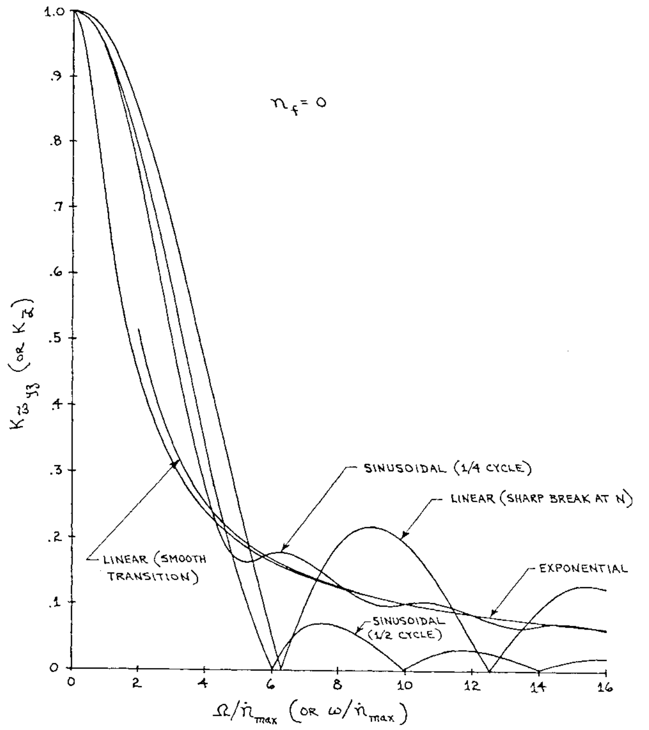

Fig. 5 Shaping effects during thrust decay.

where the initial thrust rate is

$$\dot{n}_o = \beta(1 - n_o)$$

The K-factor for the quarter-cycle sinusoidal rise with $n_o = 0.2$ is plotted in Fig. 3 as a function of $\Omega/\dot{n}_o$. The decay factor, $n_f = 0$, is plotted as a function of $\Omega/\dot{n}_f$ in Fig. 5. The corresponding pointing error factor $K_{\tilde{z}}$ is simply obtained by replacing the polhode rate Ω with the spin rate ω.

Sinusoidal Half-Cycle Decay

The previous quarter-cycle sinusoidal data is applicable to decay analyses for thrust termination at maximum slope. The half-cycle sinusoidal decay is considered in order to provide some measure of the improvement that can be achieved with zero-slope termination. The K-factor is

$$K_{\tilde{\omega}_{yz}} = 1/2(e^{i\pi(\Omega/\beta)} + 1)/[1 - (\Omega/\beta)^2] \tag{26}$$

where

$$\dot{n}_{max} = \beta/2$$

This K-factor is compared to the quarter-cycle sinusoidal decay in Fig. 5 on the basis of identical maximum slopes and therefore identical decay time intervals. The performance of the half-cycle shape is superior for large $\Omega/\dot{n}$.

Exponential Rise (Decay)

The K-factor is

$$K_{\tilde{\omega}_{yz}} = 1 + i(1 - n_o)^2/[(\dot{n}_o/\Omega) - i(1 - n_o)] \tag{27}$$

The exponential K-factor magnitude decreases monotonically toward n_o as the parameter $\Omega/\dot{n}_o$ increases. Similarly, the pointing error factor decreases monotonically with $\omega/\dot{n}_o$. Clearly, the relative pointing error will be less than the relative transverse rate for all prolate vehicles and for oblate vehicles with an inertia ratio less than two. Mathematically, for the exponential profile

$$K_{\tilde{z}} < K_{\tilde{\omega}_{yz}} \quad \text{if} \quad I_x/I < 2 \tag{28}$$

Linear Rise (Decay)

Although this profile is unattractive for most applications, it provides interesting data for comparative evaluations. The K-factor is

$$K_{\tilde{\omega}_{yz}} = n_o + i(\dot{n}_o/\Omega)[1 - e^{i(1-n_o)(\Omega/\dot{n}_o)}] \tag{29}$$

which is in agreement with the transverse rate data of Ref. 1. (The corresponding $K_{\tilde{z}}$ does not agree with the pointing error data of Ref. 1 which appears to be incorrect.) The instantaneous rise to the "initial" level N_o followed by a linear rise segment is considered to be a reasonable model for some candidate motors. However, the sharp break at maximum torque N is both difficult to achieve and undesirable (see Figs. 3 and 5). The sharp break introduces distinct phase relations which cause significant K-factor variations with small variations in $\Omega\dot{n}_o$—not a desirable design situation. Furthermore, a constraint on the precise relation of Ω and ω is required for good pointing and transverse rate control.

The contribution due solely to the linear rise can be computed by assuming a very small value of $\ddot{N}$ for the transition from $\dot{N}_o$ to $\dot{N}_{\dot{N}} = 0$. The oscillatory transverse rate magnitude is approximated by

$$\tilde{\omega}_{yz} = N_o/(I\Omega) + i\dot{N}_o/(I\Omega^2) \tag{30}$$

if $\ddot{N}$ is small compared to $\Omega\dot{N}_o$ and $\Omega^2 N_o$. Naturally, this idealized result is independent of the maximum torque since the transition is not specified. Nevertheless, a K-factor can be obtained by normalizing with respect to an arbitrary N:

$$K_{\tilde{\omega}_{yz}} = n_o + i(\dot{n}_o/\Omega) \tag{31}$$

The results shown provide a useful basis for comparison with other shapes with the same N_o and $\dot{N}_o$.

The comparative rise data, Fig. 3, show that for $\Omega/\dot{n}_o$ greater than about ten the actual shape of the transition to maximum torque is of secondary importance for reasonable shapes (sharp breaks at N excluded). Typically, this corresponds to about three cycles of polhode motion during the rise for transverse rates or three cycles of roll motion for pointing considerations. Under these conditions, even the maximum torque is a secondary parameter as indicated by the linear rise (smooth transition) data. The data for the exponential rise, Fig. 4, should provide a reasonable first estimate of the K-factor for given values of n_o and $\dot{n}_o$.

The decay data, $n_f = 0$, are presented in Fig. 5. Note that the curves for the quarter-cycle sinusoid, exponential, and linear decay (smooth transition) all converge beyond $\Omega/\dot{n}_{max}$ of about ten. The terminal flare associated with the half-cycle sinusoid is highly beneficial for values of $\Omega/\dot{n}_{max}$ greater than five.

Parameter Variations between Rise and Decay

The constant-inertia assumption for the K-factor computations is reasonable since rise and decay intervals are expected to be relatively short. However, in many applications the inertia variations and low-frequency torque variation during the relatively long time interval between rise and decay must be considered. The parameters in Eq. (1) (Ω, N, I) are assumed to be slowly varying time functions between t_r and t_d. An approximate transverse rate solution devised by R. L. West (McDonnell Douglas Astronautics-East) is appropriate: the oscillatory transverse rate remains constant in the absence of jet damping while the equilibrium transverse rate varies in accordance with the time function $N/(I\Omega)$.

Mathematically

$$\omega_{yz} = \left[\omega_{yz} - \frac{N}{I\Omega}\right]_{t=t_r} e^{-i\int_0^t \Omega(t)dt} + \frac{N(t)}{I(t)\Omega(t)}, \quad t_r < t < t_d \tag{32}$$

where the first term represents the oscillatory transverse rate due to the rise transient. Substitution of this approximate solution into the time-varying version of Eq. (1) yields the angular acceleration discrepancy

$$\Delta\dot{\omega}_{yz} = d/dt[N(t)/I(t)\Omega(t)] \tag{33}$$

An indication of the accuracy of Eq. (32) for nonzero values of Eq. (33)can be obtained as follows. An equivalent disturbance torque is defined corresponding to slow variations in $\Delta\dot{\omega}_{yz}$,

$$\Delta N(\text{equivalent}) = iI(t)\,\Delta\dot{\omega}_{yz}(t) \tag{34}$$

An improved solution, denoted by a prime, can be obtained by modifying Eq. (32) to account for this equivalent torque,

$$\omega_{yz}' = \left[\omega_{yz} - \frac{(N + iI\,\Delta\dot{\omega}_{yz})}{I\Omega}\right]_{t=t_r} e^{-i\int_0^t \Omega(t)dt} + \frac{[N(t) + iI(t)\,\Delta\dot{\omega}_{yz}(t)]}{I(t)\Omega(t)}, \quad t_r < t < t_d \tag{35}$$

The transverse rate terms,

$$i\,\Delta\dot{\omega}_{yz}/\Omega$$

in Eq. (35) are an indication of the errors in the approximate solution, Eq. (32). The error in the magnitude of the approximate transverse rate will be much less than $\Delta\dot{\omega}_{yz}/\Omega$ since the error terms are 90° out of phase with the basic solution. The angular acceleration discrepancy in the equation of motion for the improved solution, Eq. (35), is,

$$\Delta\dot{\omega}_{yz}' = d/dt[i\,\Delta\dot{\omega}_{yz}(t)/\Omega(t)] \tag{36}$$

The ratio of Eq. (36) to Eq. (33) is,

$$\Delta\dot{\omega}_{yz}'/\Delta\dot{\omega}_{yz} = -(i\dot{\Omega}/\Omega^2) + (i\Delta\ddot{\omega}_{yz}/\Omega\,\Delta\dot{\omega}_{yz}) \tag{37}$$

The previous error estimates hold whenever this ratio is small. For example, in a typical case the transverse inertia I decreases by 35% during a 20 sec burn while the other parameters remain nearly constant. The transverse rate error term at the end of burn is 1.7% of the basic solution which combines vectorially with the basic solution to yield a total error in magnitude of only 0.014%. The ratio defined by Eq. (37) is small for the case, about 0.045.

The approximate solution, Eq. (32), provides a very useful design relation for the residual transverse rate. The residual transverse rate for worst-case phasing is simply,

$$\begin{aligned}\tilde{\omega}_{yz}(\text{residual}) &= |\tilde{\omega}_{yz}(\text{rise})| + |\tilde{\omega}_{yz}(\text{decay})| \\ &= K_{\tilde{\omega}_{yz}}(\text{rise})[N/(I - I_x)\omega]_{t=t_r} \\ &\quad + K_{\omega_{yz}}(\text{decay})\,[N/(I - I_x)\omega]_{t=t_d}\end{aligned} \tag{38}$$

where the K-factors are computed for the appropriate rise and decay parameters. Other studies indicate that the residual pointing error can be treated in an analogous manner since the average pointing error remains nearly constant during the post-rise phase.

Conclusions

The results presented here indicate that extraordinary spin stabilization performance improvement can be achieved by employing a gradual thrust rise and decay. It appears that the thrust shape can be the most important design factor—particularly for applications with large thrust-line/center-of-gravity offsets. Rise and decay intervals equivalent to about three cycles of polhode motion (or roll angle motion) are sufficient to provide relative transverse rates (or pointing errors) that approach n_o for rise and are less than 10% for decay. Rocket motor design penalties, if any, for a thrust-shaped system are expected to be minor in most applications. For desirable thrust shapes the pointing error reduction (relative to the rectangular case) is greater than the transverse rate reduction for prolate vehicles and for oblate vehicles with the ratio of roll to transverse inertias less than two.

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